Lakeview Christian Academy
Summer Math Packet
For Students Entering Algebra 2

Student's Name _____

This packet is designed for you to review your Algebra 1 skills and make sure you are well prepared for the start of your Algebra 2 school year. Every topic includes an explanation, examples, hyperlinks to websites, and practice problems designed to ensure your readiness for next year. This packet is expected to be completed upon your arrival back to school and students are expected to be able demonstrate a solid understanding of these topics.

It is strongly recommended that calculators **NOT be used to complete **ALL** sections of this packet since the objective of this packet is to verify your understanding of the concepts. Please show all of your work as this will be required of you throughout the entire school year. We are really looking forward to the upcoming school year and hope you are too!*

Summer 2018

To be completed during the month of August prior to the start of the school year.

Turn this math packet into your math teacher on the first day of school for your first homework grade.

I. Orders of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations inside grouping symbols. Grouping symbols include parentheses ( ) , brackets [ ] , braces { } , radical symbols , absolute value symbols and fraction bars. If an expression contains more than one set of grouping symbols, simplify the expression inside the **innermost** set first. Follow the order of operations within that set of grouping symbols and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (Remember that a fraction bar also indicates division.)

FOURTH: Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear from left to right if they do not appear in parentheses. This is also true for addition and subtraction.)

Please **E**xcuse **M**y **D**ear **A**unt **S**ally
Parentheses **E**xponents **M**ultiplication **D**ivision **A**ddition **S**ubtraction

Example 1 Simplify.

$-4^2 + 24 \div 3 \bullet 2$ **(Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".)**

$-16 + 24 \div 3 \bullet 2$

$-16 + 8 \bullet 2$

$-16 + 16$

0

Example 2 Simplify.

$4(25 - (5 - 2)^2)$

$4(25 - (3)^2)$

$4(25 - 9)$

$4(16)$

64

Example 3 Simplify.

$3x - 4(8x + 2) + 5x$ distribute the -4 first (must multiply the 8x and +2)

$3x - 32x - 8 + 5x$ combine all like terms (take the sign in front, ex: -32x-8x)

$-24x - 8$

Practice Problems for Topic I:

Simplify each expression using the orders of operations, *without* a calculator.

1.) $6+9\bullet 3$

2.) $4(2+3)^2$

3.) $\dfrac{3(4+11)}{10-3\bullet 3}$

4.) $47-3[6+8\div 2]$

5.) $\dfrac{3\bullet 2-3\bullet 1+14\div 2}{6\div 3+6\div 2}$

6.) $2(5-2)^2+2(4-2)^2$

Evaluate each expression if a = 2, b = 3, c = -2, d = 4, f = -5, *without* a calculator.

7.) $-ca^2$

8.) $(b+cd)(ac)$

9.) $(cf)-(ab)$

10.) dc^a

11.) a^2b^2

12.) $a-b-c-d$

Simplify each of the following.

Hint – remember to use orders of operations and to combine like terms (refer to section VII: properties of exponents, for #24, 25 and 27)

13.) $3x - 4 + 7x - 8 - 10x - 2$

14.) $4x + 2(x + 5)$

15.) $3(x + 5) - 4(x - 6)$

16.) $5x^3 + 2x^2 - 7x - x^3 + 5x^2 - 18$

17.) $(5x^2 + x - 4) - (9x^2 - 4x - 11)$

18.) $12x^3(x^4 - 5x^2 + 2)$

19.) $(x + 2)(x^2 - 5x + 1)$

20.) $5xy - 20x + 6y - (10xy + 10x)$

21.) $2x(3x^2 - 4x + 2) - 5x(2x + 3)$

22.) $5x^2y^2 + 3xy - 3x^2y^2 + 2x^3y^2 + 5xy$

II: Solving Linear Equations

Solving with a variable on only one side of an equations:

Ex) $-2x + 7 = 15$ ***subtract 7 from both sides***

$-2x = 8$ ***divide -2 from both sides***

$x = -4$

$-2(-4) + 7 = 15$ *check your solution using substitution*

$8 + 7 = 15$

$15 = 15$

Solving with a variable on both sides of the equation:

Ex) $8x – 10 = 2x + 14$ ***Move the smaller x value to the other side***

$6x – 10 = 14$ ***(subtracted 2x from both sides)***

$6x = 24$ ***add 10 to both sides (your non x variable)***

$x = 4$ ***divide both sides by 6***

$8(4) – 10 = 2(4) + 14$ ***check by substitution***

$32 -10 = 8 + 14$

$22 = 22$

Solve using the properties of real numbers:

Ex) $-3(2x + 5) + 6x = 11x + 7$ ***Use the distributive property***

$-6x - 15 + 6x = 11x + 7$ ***Combine like terms (on the same side of the equation)***

$-15 = 11x + 7$ ***subtract 7 from both sides***

$-22= 11x$ ***divide both sides by 11***

$-2 = x$

$-3(2(-2) + 5) + 6(-2) = 11(-2) + 7$ *check by substitution*

$-3(-4+5) - 12 = -22 + 7$

$-3(1) - 12 = -15$

$-15 = 15$

Practice Problems for TOPIC II:

Solve the following equations and check your solutions.

1.) $-3x + 4 = 11$

2.) $\frac{1}{2}x - 8 = 3$

3.) $\frac{2}{3}x + 6 = 18$

4.) $6 - x - 5 = -4x - 3 - x$

5.) $-5 + \frac{b}{4} = 7$

6.) $4.2m + 4 = 25$

7.) $4t + 7 + 6t = -33$

8.) $6 = -z - 4$

9.) $\frac{1}{3} + \frac{4}{6}y = \frac{2}{3}$

10.) $6(y - 2) = 8 - 2y$

11.) n + 3(n – 2)=10.4

12.) $\frac{m}{12} = 2.7$

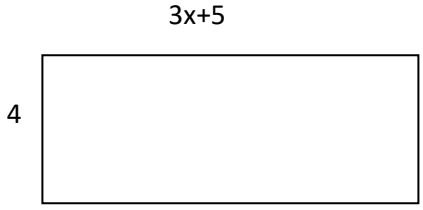
13.) Solve 2x -4 = -7. Justify each step.

14.) A taxicab company charges each person a flat fee of $1.85 plus an additional $.40 per quarter mile.

A. Write a formula to find the cost for each fare.

B. Use the formula to find the cost for 1 person to travel 8 mi.

15.) Find the dimensions of the rectangle given the area = 164 sq. ft.



III: Using Slope and y-Intercept to Graph Linear Equations and to Write Equations of Lines

There are 3 forms to write an equation of a line:

Slope intercept form: $y = mx + b$ m = slope, b = y intercept

Point slope form: $(y-y_1) = m(x-x_1)$ (x_1, y_1) is a point on the line

Standard form: $Ax + By = C$

y intercept: where x = 0 **x intercept**: where y = 0

**to find these values plug either 0 in for x or 0 in for y and solve for the other value*

I.) Graphing in slope intercept form, y = mx + b:

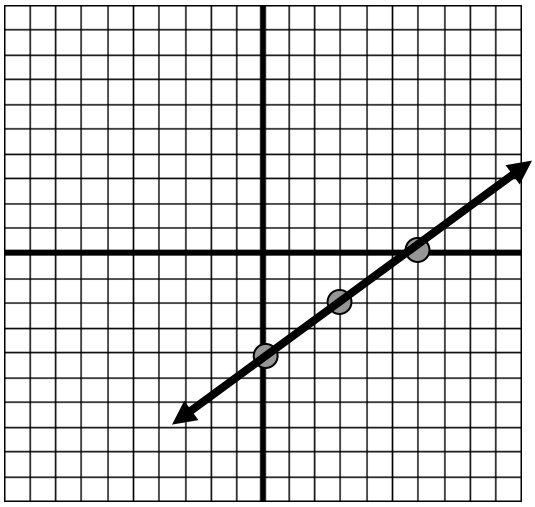
Ex. 1 $y = \frac{2}{3}x - 4$

Step 1: Identify "b" (0,-4)

Step 2: Plot "b" on the grid

Step 3: Use the slope to plot your next point (rise/run) starting from the y-intercept (3, -2), (6, 0)

Step 4: Continue step three until at least 3 points are plotted.



Ex. 2

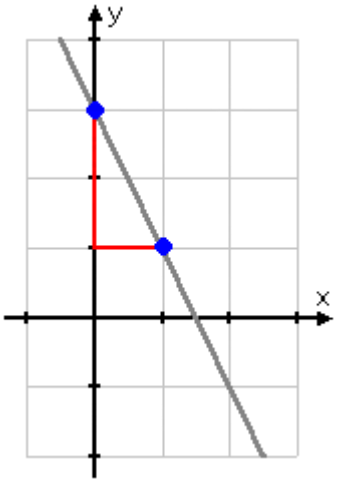
$y = -2x + 3$

1.) Plot "b" (0,3)

2.) Plot your slope (-2/1)

Down 2 and right 1

3.) Connect your points

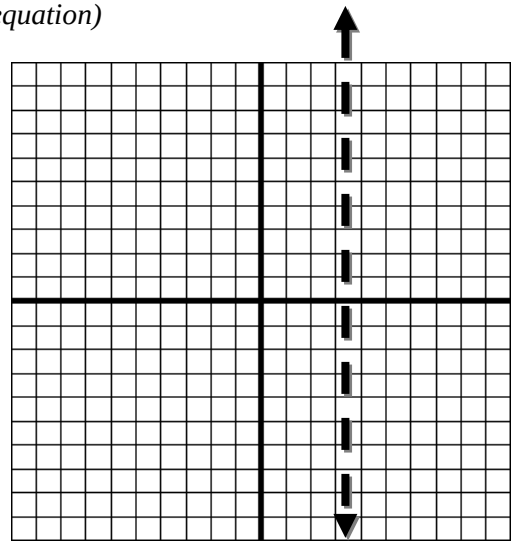


(line crosses through both the 'x' and 'y' axes, hence the 'x' and 'y' variable in the equation)

Ex. 3

X = 3 ***This is a vertical line with an undefined slope***

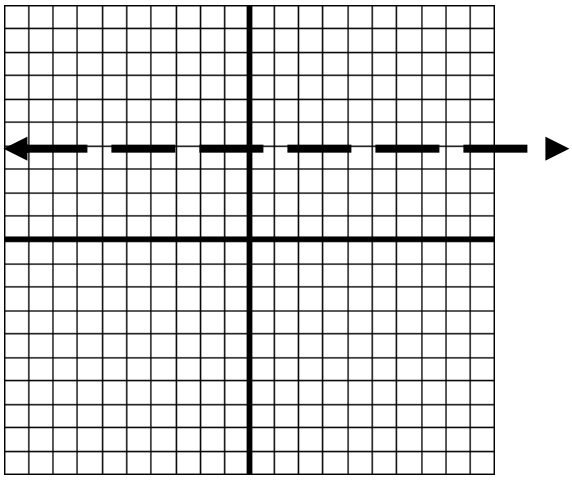
(line only crosses through x-axis, hence it only has the 'x' variable in the equation)



Ex. 4

y = 4 ***This is a horizontal line with a slope = 0***

(line only crosses through y-axis, hence it only has the 'y' variable in the equation)



II.) Writing equations of lines in slope intercept form:

Ex. 1 – Write an equation of a line that passes through the point (2, 1) and has a slope of 3.

y = mx + b	plug 3 in for m, 2 in for x, and 1 in for y [every ordered pair is (x, y)]
1 = 3(2) + b	simplify
1 = 6 + b	solve for b by subtracting 6
-5 = b	
y = 3x – 5	plug 3 in for m and -5 in for b

Given two points (x_1, y_1) and (x_2, y_2), the formula for the slope of the straight line going through these two points is:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Ex. 2 - Write an equation of a line that passes through the points (2, 4) and (5, 6).

$\frac{6-4}{5-2} = \frac{2}{3}$	start by finding slope using the formula above
m = $\frac{2}{3}$; (2 , 4)	plug the slope and one of the points (doesn't matter which point since the line passes through both points) into the Point Slope Formula or slope-intercept formula (which ever formula you prefer to use)
$y - 4 = \frac{2}{3}(x - 2)$	simplify by distributing the 2/3
$y - 4 = \frac{2}{3}x - \frac{4}{3}$	add 4 to both sides to get 'y' by itself
y = 2/3 x + 8/3	

Parallel Lines: two lines that have the same slope and never intersect

Ex. y = 2x + 3 is parallel to y = 2x -1

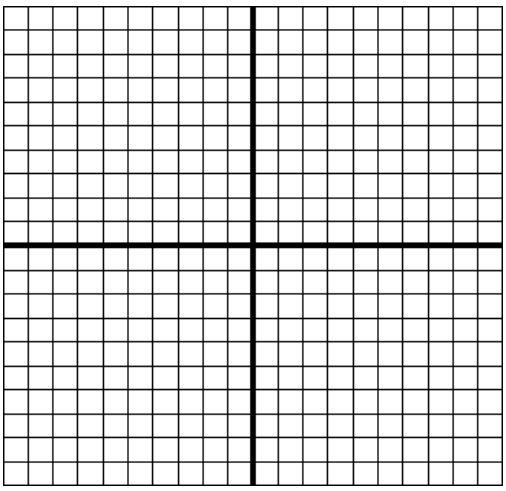
Perpendicular Lines: two lines whose slopes are opposite reciprocals

Ex. y = 2x + 3 is perpendicular to y = -1/2 x **2 and* $\frac{-1}{2}$ *are opposite reciprocals*

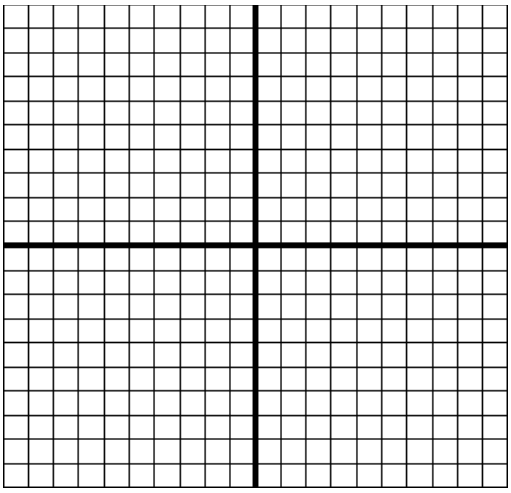
Practice Problems for Topic III.

Sketch each of the lines on the graph below.

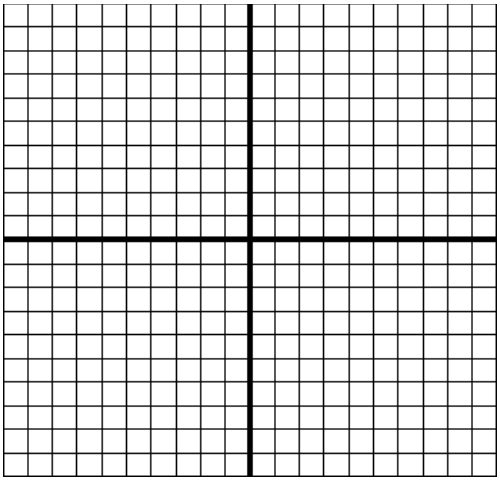
1. y = 2x



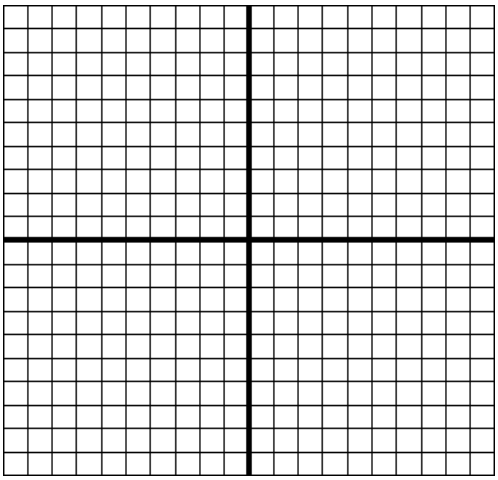
2. y = -3x



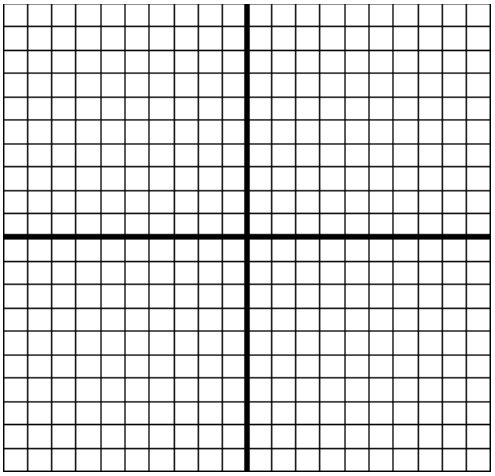
3. $y = \frac{5}{4}x$



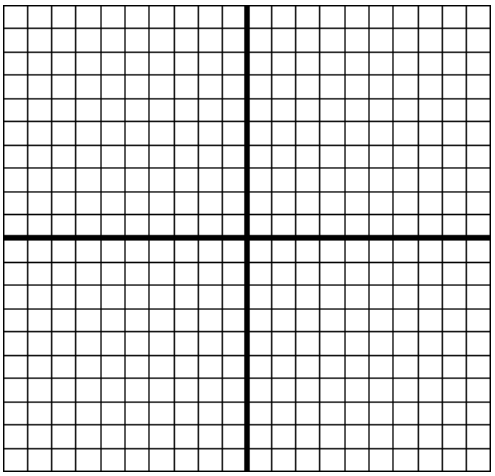
4. $y = -\frac{4}{3}x$



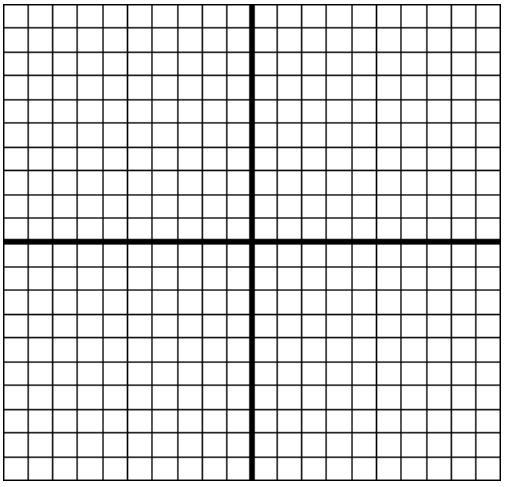
5. y = 6



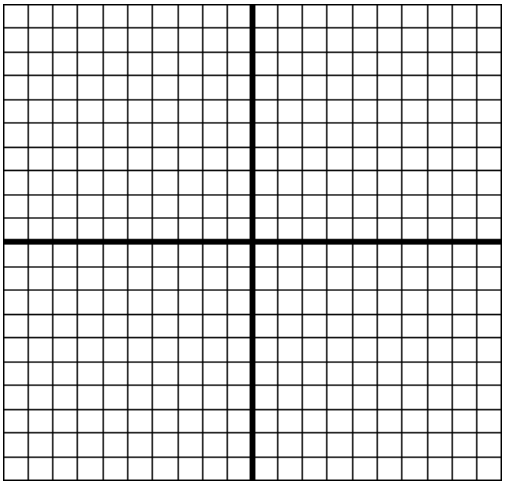
6. x = -4



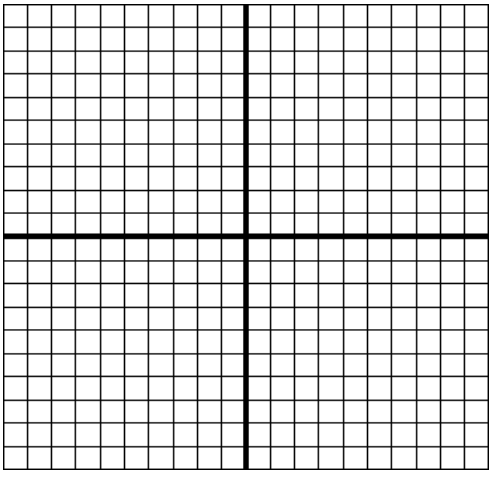
7. $y = \frac{4}{5}x - 6$



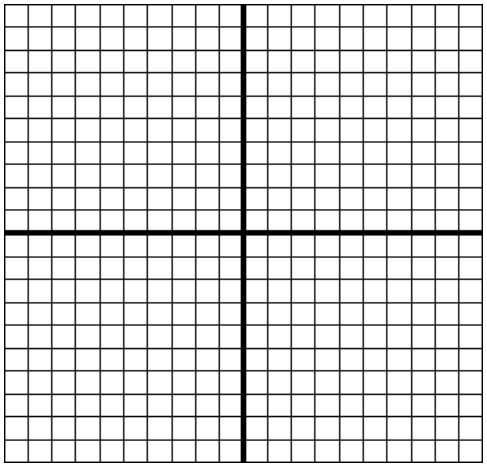
8. $y = -\frac{5}{3}x + 8$



9. 2x + 3y = 12

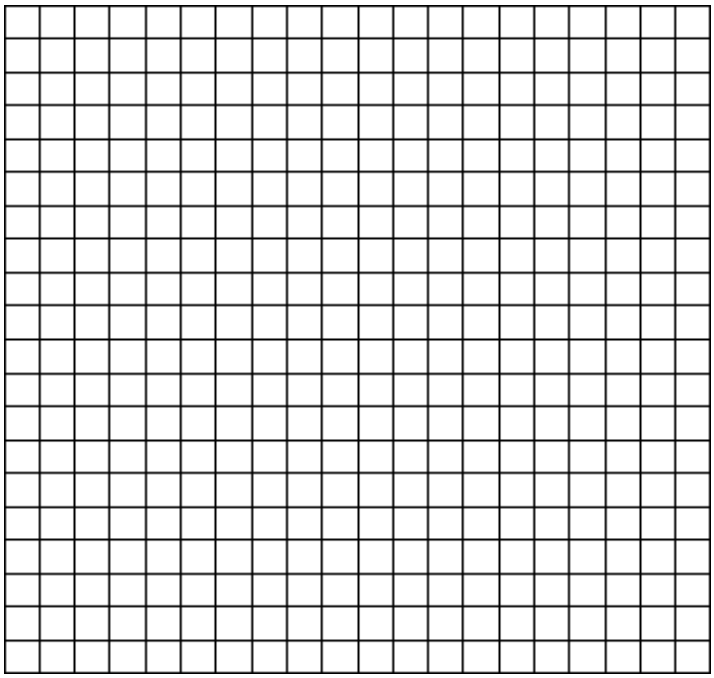


10. -3x + 27y = 8



11. Complete the table and graph the points on the graph below.

x	2x + 3
-4	
-2	
0	
2	
4	
	-15
	25



12. A hot air balloon is currently at an altitude of 10,000 feet. The pilot begins to descend the balloon at a rate of 50 feet per minute.

A. Write an equation for the altitude (a) of the balloon as a function of the time (t)

B. Find the altitude of the balloon after:

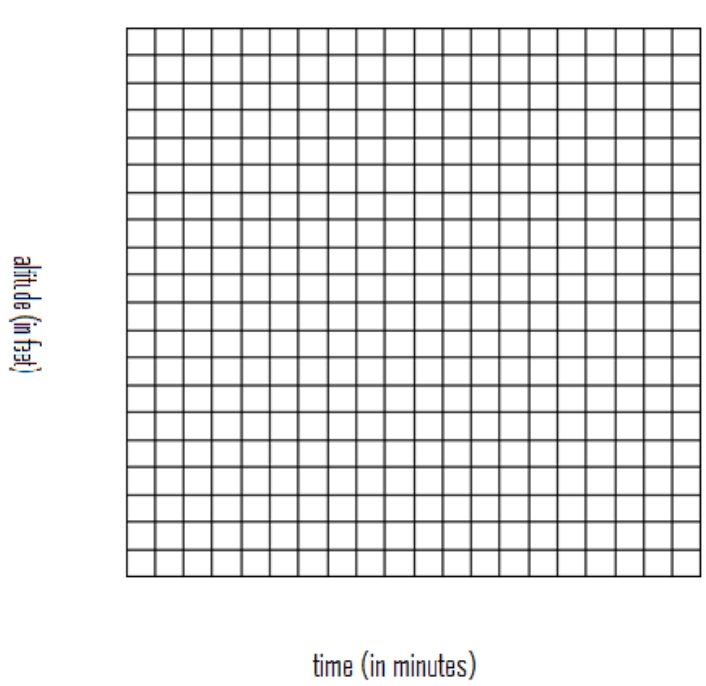
a. 10 minutes b. 30 minutes c. 1 hour

C. How long will it take for the balloon to be:

a. 7000 feet b. 2000 feet c, on the ground

D. Complete the table, and graph the altitude of the balloon as a function of the time in minutes that the balloon takes to begin its descent and land on the ground.

time (in minutes)	altitude (in feet)
0	7000
10	2000
30	0
60	



13. This summer you and a friend recorded your first CD. In order to produce your CD you need to hire a recording studio. One studio you called charges $250 for making a master CD and $3 to burn each CD after that.

(a.) Write an equation to model the cost of burning the CDs.

_____

(b.) How much will it cost to burn 75 CDs? _____

(c.) How many CD's can be made for $3000? _____

More practice for Topic III.

1. $y = 4x - 2$

slope = _____

y-intercept (let x = 0) = _____

x-intercept (let y = 0) = _____

2. $6x - 3y = 15$

slope = _____

y-intercept (let x = 0) = _____

x-intercept (let y = 0) = _____

3. $y = 2x - 5$

slope = _____

y-intercept (let x = 0) = _____

x-intercept (let y = 0) = _____

4. $4x + 3y = 12$

slope = _____

y-intercept (let x = 0) = _____

x-intercept (let y = 0) = _____

5. $3y = 4x - 9$

slope = _____

y-intercept (let x = 0) = _____

x-intercept (let y = 0) = _____

6. $5x - y = 15$

slope = _____

y-intercept (let x = 0) = _____

x-intercept (let y = 0) = _____

Find the slope and y-intercept for each line given below. Then write the equation in slope-intercept (y = mx+b) form.

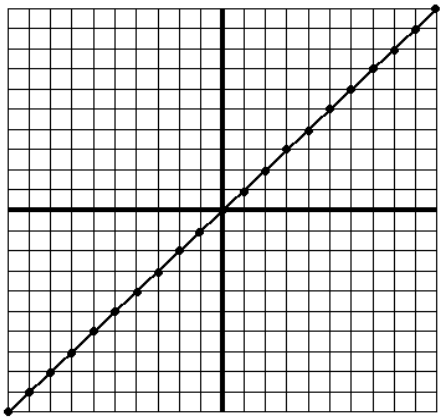
7. A line that passes through the point (-5, 6) with a slope of 4.

8. A line that passes through the points (4, 1) and (7, -11)

9. A line is perpendicular to the line in #2 above and passes through the x-axis at 3.

Find the slope and y-intercept and x-intercept of each line. Then write the equation of each line in y = mx + b form.

10.



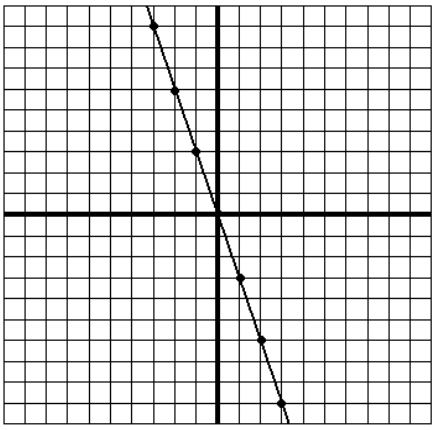
Slope = _____

y-intercept : _____

equation: y = _____

x-intercept: _____

11.



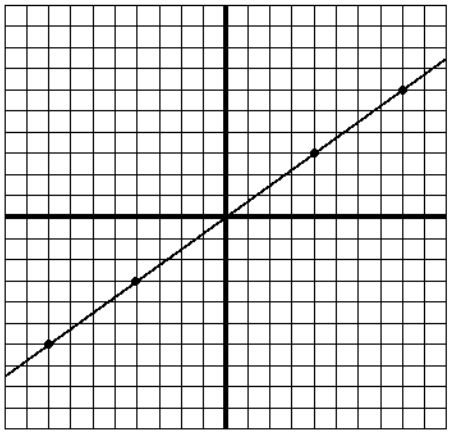
Slope = _____

y-intercept: _____

equation: y = _____

x-intercept:_____

12.



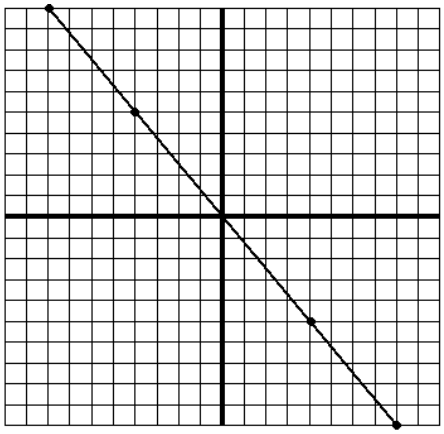
Slope = _____

y-intercept : _____

equation: y = _____

x-intercept: _____

13.



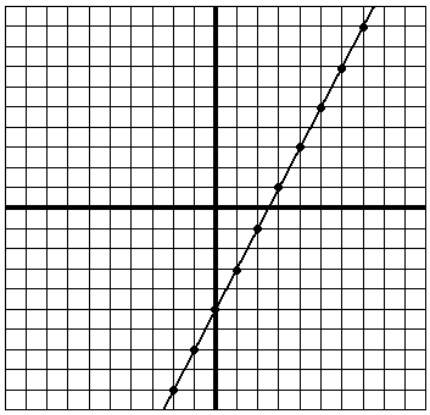
Slope = _____

y-intercept: _____

equation: y = _____

x-intercept:_____

14.



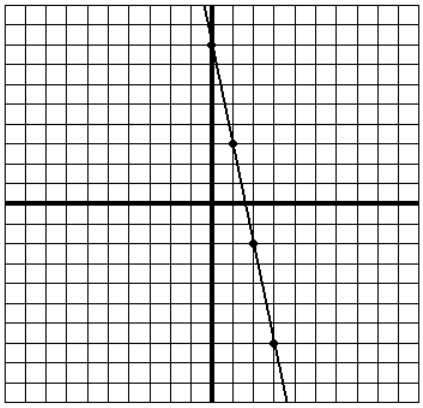
Slope = _____

y-intercept : _____

equation: y = _____

x-intercept: _____

15.



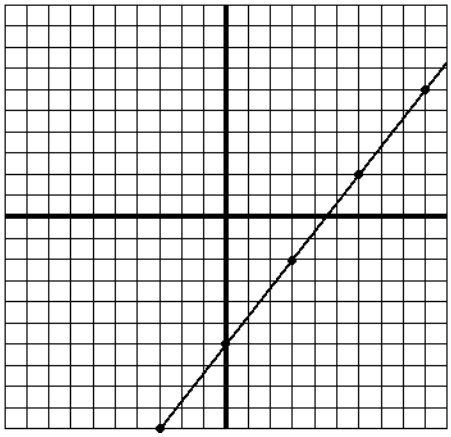
Slope = _____

y-intercept: _____

equation: y = _____

x-intercept:_____

16.



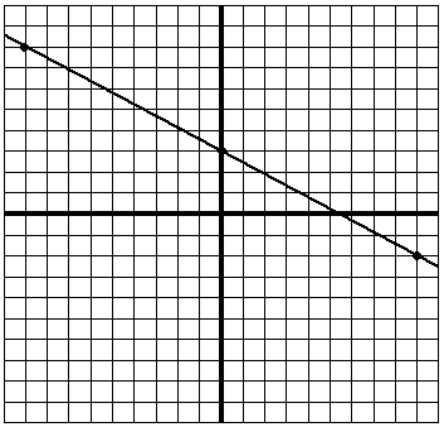
Slope = _____

y-intercept : _____

equation: y = _____

x-intercept: _____

17.



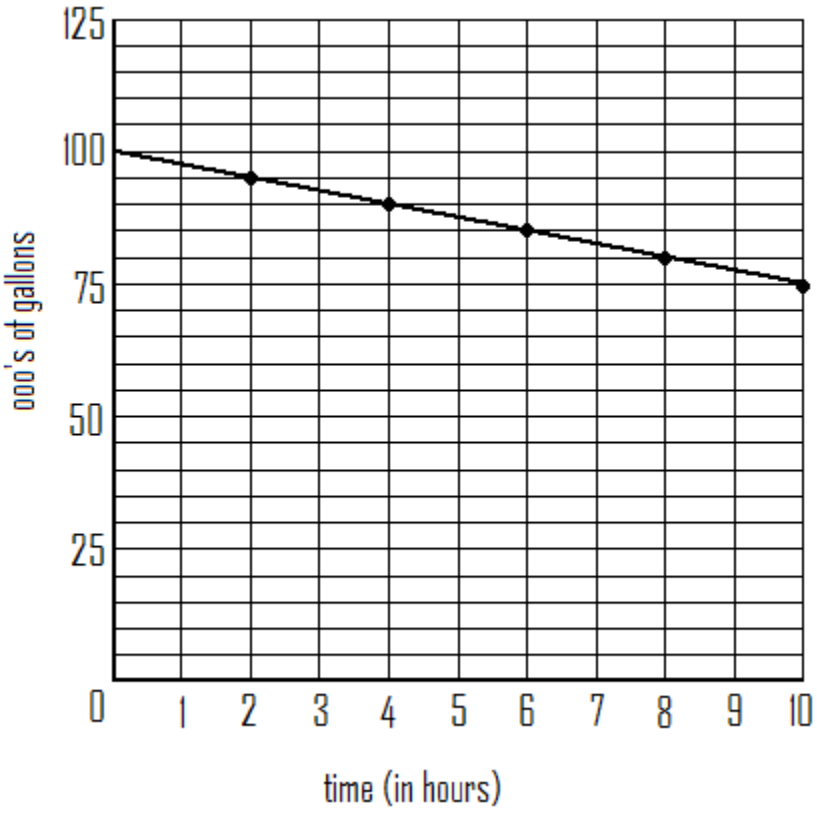
Slope = _____

y-intercept: _____

equation: y = _____

x-intercept:_____

18. A tanker that is filled with oil pulls into an oil refinery. The tanker is going to offload the oil as shown in the graph below.



A. How many gallons of oil are in the tanker when it pulls into the refinery? _____

B. At what rate, in gallons per hour, is the tanker unloading the oil? _____

B. Write an equation for the amount of oil (l) after h number of hours: l = _____

D. Find the amount of oil after:

a. 15 hours b. 25 hours

E. How long will it take for the tanker to have:

a. 60,000 gallons left b. 25,000 gallons left

IV. Linear Systems

System of equations: a set of two or mre equations that use the same variables.

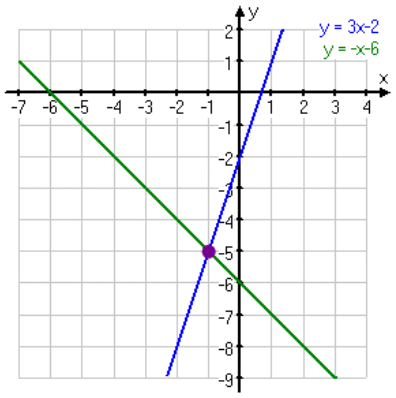
Solution to a system: the point of intersection (x, y)

3 methods to solving a system:

1. Graphing
2. Substitution
3. Elimination

Solving a System by Graphing

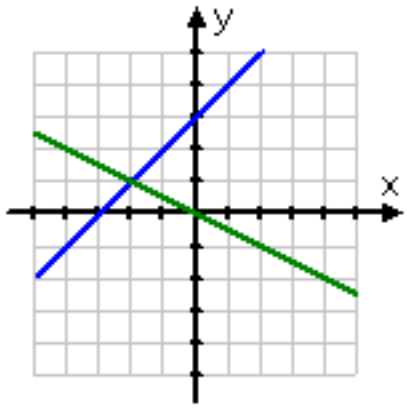
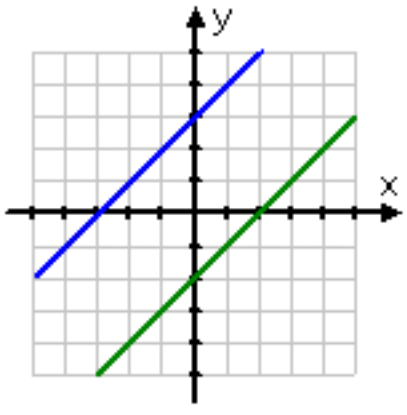
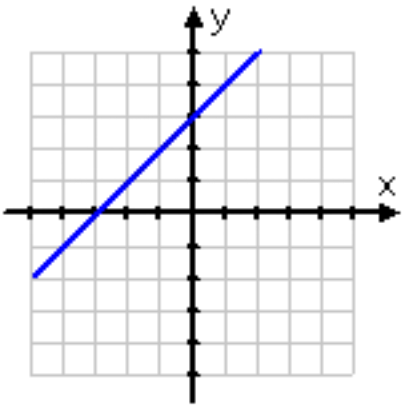
Ex 1. $y = 3x - 2$
$y = -x - 6$



1. *Graph both lines*
2. *Find where they intersect*

Solution: (-1, -5)

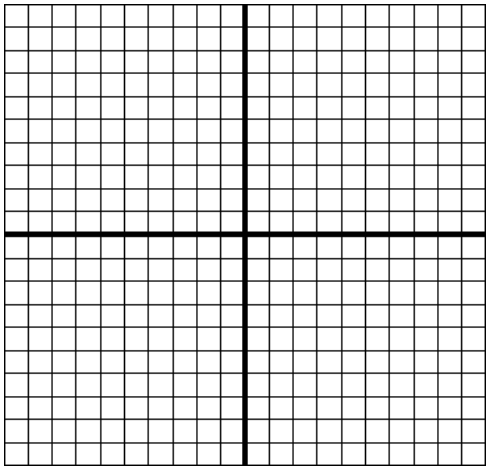
When you are solving systems, you are finding the intersection of lines. For two-variable systems, there are three possible types of solutions:

Independent system: one solution and one intersection point	Inconsistent system: no solution and no intersection point	Dependent system: the solution is the whole line
		
Solution: (x, y)	**Solution: No Solution**	**Solution: Infinite Solutions**

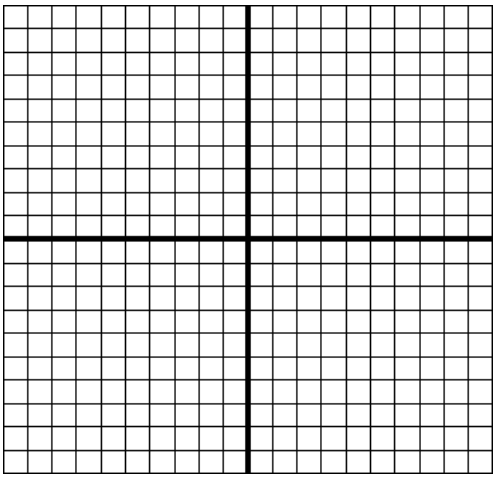
Practice Problems for Topic IV.

Solve the following systems by graphing.

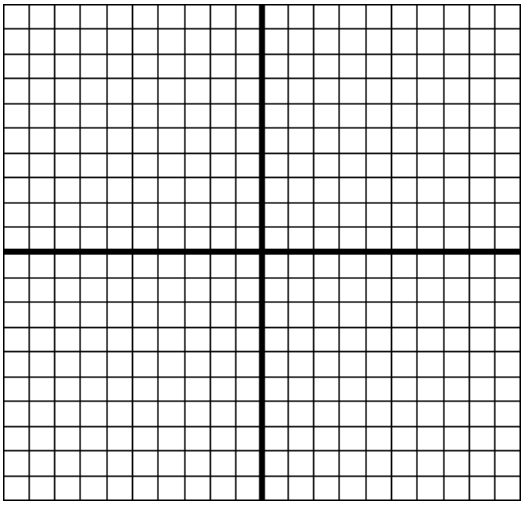
1. $y = 2x + 1$
   $y = 3x - 2$



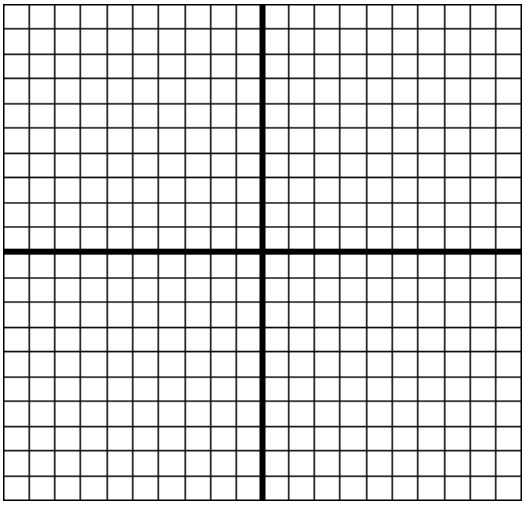
2. $y = \frac{1}{2}x - 7$
   $y = 4x$



3. $y = 3x + 2$
   $y = 3x - 6$



4. $y = -x + 3$
   $y = \frac{1}{2}x + 6$



Solving Systems using Substitution

Ex 1. Solve $\begin{cases} y = 3x - 2 \\ y = -x - 6 \end{cases}$

3x – 2 = -x – 6	Set the equations equal to each other and solve for x
4x = 8	
x = 2	
y = 3(2) – 2	Now plug 2 in to one of the equations for x and solve for y
y = 4	

Solution: (2, 4)

Ex 2. Solve $\begin{cases} x = 3y + 2 \\ 2x + y = 11 \end{cases}$

2(3y + 2) + y = 11	Substitute 3y + 2 for x in 2x + y = 11 and solve for y
6y + 4 + y = 11	
7y = 7	
y = 1	
x = 3(1) + 2	Plug in 1 for y in one of the equations and solve for x
x = 5	

Solution: (5, 1)

More Practice for Topic IV.

Solve the following systems using substitution.

1. $\begin{cases} y = 7x - 1 \\ y = 3x + 11 \end{cases}$

2. $\begin{cases} y = 0.5x \\ y = 5x - 9 \end{cases}$

3. $\begin{cases} x = y - 1 \\ x + y = 9 \end{cases}$

4. $\begin{cases} y = -3x + 5 \\ 2x - 2y = 14 \end{cases}$

Solving Systems using Elimination

The Elimination Method requires that you eliminate a variable to be able to solve the system.

In order to eliminate a variable when you add the equations together, there are 2 qualifications:

1. The variables must have the same coefficient (same number in front of the variable).
2. They must also have opposite signs.

   **Always add the equations

Solve the following systems using elimination.

Ex 1. $x + y = 12$
$x - y = 14$

**Since the 'y' variable meets both requirements(has same coefficient and opposite signs), we can just add the equations together.

2x = 26

x = 13

13 + y = 12 Plug in 13 for x in one of the equations and solve for y

y = -1

Solution: (13, -1)

Ex 2. $-2x + y = 2$
$2x + y = 6$

**Since the 'x' variable meets both requirements, we can just add the equations together.

2y = 8

y = 4

-2x + 4 = 2 Plug in 4 for y in one of the equations and solve for x

-2x = -2

x = 1

Solution: (1, 4)

If the variable(s) do not meet both requirements, you multiply one or both of the equations so that it will satisfy both qualifications.

Ex 3. $3x - y = 5$
$2x - 2y = -2$

**You can eliminate either variable, but I choose to eliminate the 'y' variable because I will only have to multiply the top equation.

-2 (3x – y = 5) I multiplied by -2 to make them have opposite signs and same coefficient

2x – 2y = -2

The new system is:

-6x + 2y = -10

2x – 2y = -2 Now I can add the equations together and solve for x

-4x = -12

x = 3

3(3) – y = 5 Plug in 3 for x in one of the **original** equations and solve for y

9 – y =5

-y = -4

y = 4

Solution: (3, 4)

TOPIC V: Power Properties

Multiplication: Recall $(x^m)(x^n)=x^{(m+n)}$ Ex: $(3x^4y^2)(4xy^5)=(3\cdot 4)(x^4\cdot x^1)(y^2\cdot y^5)=12x^5y^6$

Division: Recall $\frac{x^m}{x^n}=x^{(m-n)}$ Ex: $\frac{42m^5j^2}{-3m^3j}=\left(\frac{42}{-3}\right)\left(\frac{m^5}{m^3}\right)\left(\frac{j^2}{j^1}\right)=-14m^2j$

Powers: Recall $(x^m)^n = x^{(m\cdot n)}$ Ex: $(-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}$

Power of Zero: Recall $x^0 = 1,\ x \neq 0$ Ex: $5x^0y^4 = (5)(1)(y^4) = 5y^4$

PRACTICE

Simplify each expression.

1. $(c^5)(c)(c^2)$
2. $\frac{m^{15}}{m^3}$
3. $(k^4)^5$
4. d^0
5. $(p^4q^2)(p^7q^5)$
6. $\frac{45y^3z^{10}}{5y^3z}$
7. $(-t^7)^3$
8. $3f^3g^0$
9. $(4h^5k^3)(15k^2h^3)$
10. $\frac{12a^4b^6}{36ab^2c}$
11. $(3m^2n)^4$
12. $(12x^2y)^0$
13. $(-5a^2b)(2ab^2c)(-3b)$
14. $4x(2x^2y)^0$
15. $(3x^4y)(2y^2)^3$

VI. Quadratics

<u>Multiplying Binomial Expressions</u>: F.O.I.L. (First, Outer, Inner, Last)

FOIL is the method by which two binomials can be multiplied using the distributive property.

Example: Multiply the binomials using FOIL.

$(x+3)(x+2)$ Take the first term of the first binomial and distribute it through the second binomial. Then, take the second term in the first binomial and distribute it through the second binomial.

$= (x \bullet x) + (x \bullet 2) + (3 \bullet x) + (3 \bullet 2)$

$= (x^2) + (2x) + (3x) + (6)$

$= (x^2 + 5x + 6)$

Example: $(3x + 4)^2$ Keep in mind that when you square something, you are multiplying it by itself

$= (3x+ 4)(3x + 4)$

$= (3x \bullet 3x) + (3x \bullet 4) + (4 \bullet 3x) + (4 \bullet 4)$

$= (9x^2) + (12x) + (12x) + (16)$

$= 9x^2 + 24x + 16$

Practice Problems for Topic VI:

Foil:

1.) (x – 2)(x + 6)

2.) (3x + 6)(x – 1)

3.) (x + 9)(-2x – 7)

4.) (5x + 1)(3x – 2)

5.) (x – 8)(2x + 3)

6.) (5n + 6)(5n – 3)

7.) (7a – 6)(8a + 2)

8.) $(5a – 3)^2$

9.) (2x + 5)(-3x + 5)